2018

(October)

MATHEMATICS

(Honours)

(Differential Equations and Advanced Dynamics)

(GHS–52)

Marks : 75

Time : 3 hours

The figures in the margin indicate full marks for the questions

Write Units I and II together in one answer script and Units III, IV and V together in another answer script

Answer **five** questions, choosing **one** from each Unit

UNIT—I

1. (a) Solve : 4

$$\frac{d^2y}{dx^2}+3\frac{dy}{dx}+2y=e^{2x}\sin x$$

(Turn Over)

(b) Solve the equation

$$x\frac{d^2y}{dx^2}-\frac{dy}{dx}+4x^3y=x^5$$

by changing the independent variable. 6

(c) Solve : 5

$$\frac{d^2y}{dx^2}+\frac{1}{x}\frac{dy}{dx}=\frac{12\log x}{x^2}$$

2. (a) Test for exactness and solve

$$(1+x^2)\frac{d^2y}{dx^2}+4x\frac{dy}{dx}+2y=\sec^2 x$$

given that $y=0$, $\frac{dy}{dx}=1$ when $x=0$. 6

(b) Solve the simultaneous equation : 4

$$\frac{dx}{mz-ny}=\frac{dy}{nx-lz}=\frac{dz}{ly-mx}$$

(c) Show that the equation

$$(yz+xyz)dx+(zx+xyz)dy+(xy+xyz)dz=0$$

is integrable and find its solution. 5

UNIT—II

(In this Unit, $p=\frac{\partial z}{\partial x}$, $q=\frac{\partial z}{\partial y}$)

3. (a) Form a partial differential equation by eliminating the arbitrary function ϕ from $\phi(x+y+z, x^2+y^2-z^2)=0$. What is the order of this partial differential equation? 6

(Continued)

4

(b) Solve :

$$x(y^2+z)p-y(x^2+z)q=z(x^2-y^2)$$

(c) Find complete and singular integrals by Charpit's method of

$$2xz-px^2-2qxy+pq=0$$ 3+2

4. (a) Find the equation of the integral surface satisfying the differential equation $4yzp+q+2y=0$ and passing through $y^2+z^2=1$, $x+z=2$. 6

(b) Prove that the complete integral of $z=px+qy-2p-3q$ represents all possible planes through the point (2, 3, 0). Also find the envelope of all planes represented by the complete integral. 6

(c) Find the complete integral of $p^2+q^2=m^2$ where m is a constant. 3

UNIT—III

5. (a) A particle slides in a vertical plane down a rough cycloidal arc whose axis is vertical and vertex downwards, starting from a point, where the tangent makes

(Turn Over)

an angle θ with the horizon and coming to rest at the vertex. If μ be the coefficient of friction, show that

$$\mu(\sec\theta \cdot e^{\mu\theta} + 1) = \tan\theta$$ 8

(b) A particle moves under a central force $\mu(3a^3u^4 + 8au^2)$ and is projected from an apse at a distance a from the centre of force with velocity $\sqrt{10\mu}$. Show that the second apsidal distance is half of the first. 7

6. *(a)* A particle is describing an ellipse of eccentricity e about a centre of force at a focus. Prove that—

(i) $rav^2 = \mu(2a - r)$

(ii) $h^2 + \mu a e^2 = \mu a$

with the usual notations. 3+2

(b) If a particle P slides down a rough cycloid whose axis is vertical and vertex lowest, then show that

$$v^2 = \frac{4ag}{1+\mu^2}[A^2 e^{2\mu\theta} - (\sin\theta - \mu\cos\theta)^2]$$

where μ is the coefficient of friction, θ the angle made by the tangent at P and A being any constant depending on initial conditions. 10

UNIT—IV

7. *(a)* Define the term 'equimomental systems'. State and prove the necessary and sufficient conditions for two systems to be equimomental. 2+8

(b) Find an equimomental system of particles for a uniform rod AB of mass M. 5

8. *(a)* Show that a uniform solid cuboid of mass M is equimomental with—

(i) masses $\frac{1}{24}M$ at the midpoints of its edges and $\frac{1}{2}M$ at its centre;

(ii) masses $\frac{1}{24}M$ at its corners and $\frac{2}{3}M$ at its centre. 5+5

(b) A square of side a has particles of masses m, $2m$, $3m$ and $4m$ at its vertices. Show that the principal moments of inertia at the centre of the square are $2ma^2$, $3ma^2$, $5ma^2$ and find the directions of the principal axes. 5

UNIT—V

9\. *(a)* A uniform rod AB of length $2a$ can turn freely about one end A, and at time $t = 0$ is hanging from A in equilibrium under gravity. The end A is then set in motion in a horizontal straight line so that when $t \geq 0$, $OA = vt + \frac{1}{2}ft^2$, where O is a fixed point in the line and v, f are constants. Show that the initial angular velocity of the rod is $\frac{3v}{4a}$ and it will make complete revolutions about A, if

$$3v^2 > 8a[g + (g^2 + f^2)^{1/2}]$$ 9

(b) A uniform circular disc of mass M and radius a is rotating in its plane with initial angular velocity ω, its centre being at rest. If a point on the rim be suddenly fixed, find the new angular velocity of the disc and the velocity of its centre. 6

10\. *(a)* A uniform rod AB of mass M and length $2a$ lies at rest on a smooth horizontal table. An impulse J is applied at A in the plane of the table and perpendicular to the rod. Determine the velocity of the centroid and the angular velocity of the rod. 7

(Continued)

(b) A uniform rod of mass m and length $2a$ falls from rest in a vertical position with one end fixed on a table which is so rough that slipping never occurs. Show that when the rod is inclined to the vertical at an angle θ, the angular velocity is $\sqrt{\frac{3g}{a}}\sin\left(\frac{\theta}{2}\right)$ and that the rod will leave the table when the inclination to the vertical is $\cos^{-1}\left(\frac{1}{3}\right)$. 8

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